Waves

A wave is a periodic disturbance which travels with finite velocity through a medium and remains unchanged in type as it travels. Or it is a disturbance which travels through a medium, transforms energy from one location to another without transferring matter.

Waves may be classified as mechanical or electromagnetic waves.

Mechanical waves require a material medium for their propagation. These include water waves, sound waves and waves on stretched strings.

Electromagnetic waves include radio, infra red, light, Ultraviolet, X-rays, Gamma rays. Electromagnetic waves travel in a vacuum.

If the disturbance of the source of waves is simple harmonic, the displacement in a given time varies with distance from the source as shown below.

Amplitude: This is the greatest displacement of any wave particle from its equilibrium position.

Wavelength(λ):the distance between two successive crests or troughs. Or it is the distance between two consecutive wave particles in phase.

Period(T): The time taken for any particle to undergo a complete oscillation.

Frequency(f): The number of oscillations per second.

Relationship between f and T

If a wave completes n cycles in time t, then frequency, $f = \frac{n}{t}$ .... (i)

Period $T = \frac{t}{n}$ .... (ii)

Eqn (i) x eqn (ii)

$$fT = 1$$

$$f = \frac{1}{T}$$

Velocity(v): The distance covered by a wave particle per second.

Relationship between v, λ and f

If a wave of wavelength λ completes n cycles in time t, then frequency

Total distance covered = nλ

Speed, $v = \frac{distance}{time} = \frac{n\lambda}{t} = \frac{n}{t} \times \lambda$

But $\frac{n}{t} = f$

$v = f\lambda$

Alternatively,

F a wave covers a distance, λ, the wavelength, then the time taken is T, the period. Hence

speed , $v = \frac{\lambda}{T}$

But $f = \frac{1}{T}$. Hence $v = \lambda f$

Types of waves

Transverse Waves: It is one which propagates by vibrations perpendicular to the direction of travel of the wave. Examples include: Water waves, electromagnetic waves and waves of a stretched string.

Speed of a transverse waves along a stretched strings

The speed v of a transverse waves on a stretched string is independent of the amplitude and frequency of the wave. It depends on the tension, T in the string and the mass per unit length μ. The tension will determine the restoring force on a displaced piece of string and mass per unit length will effect its consequent acceleration.

Using dimension analysis

$v = kT^{x}\mu^{y}$, k is a dimensionless constant.

$[v] = [T^{x}\mu^{y}]$

$LT^{-1} = (MLT^{-2})^{x}(ML^{-1})^{y}$

Equating power, you get $x = \frac{1}{2}, y = -\frac{1}{2}$

Hence $v = kT^{\frac{1}{2}}\mu^{\frac{-1}{2}}$

Further analysis gives k = 1.

Hence $v = \sqrt{\frac{T}{\mu}}$

Longitudinal waves

The vibrations of the individual particles occur in the same direction as the direction of the travel of the wave. Examples include sound waves and waves in a stretched spring.

The speed of longitudinal waves is given by $v = \left(\frac{Y}{\rho}\right)^{\frac{1}{2}}$ where Y is young'*s modulus and* ρ is the mass density of a solid.

For fluids, the expansion for the speed of longitudinal wave is given by $v = \left(\frac{\beta}{\rho}\right)^{\frac{1}{2}}$.

$\beta = -\frac{V\Delta P}{\Delta V}$ is the adiabatic bulk modulus and ρ is the density of fluid.

For gases in particular, $v = \left(\frac{\gamma P}{\rho}\right)^{\frac{1}{2}}$ Where P is pressure, ρ is density and γ is the ratio of molar heat capacities.

Progressive wave/ traveling waves

A progressive wave consists of a disturbance moving from a source to the surrounding places as a result of which energy is transferred from one point to another.

Both transverse and longitudinal are progressive waves. The profile of a progressive wave moves along the speed of the wave. It repeats itself at equal distances. The repeat distance is called the wavelength.

If one point in the medium in which the profile propagates is taken, the profile is seen to repeat itself at equal intervals of time called the period. Vibrations of particles in progressive waves are of the same amplitude and frequency but the phase of the vibration changes for different points along the wave.

Phase difference

When the crests of two waves of equal wavelength are together, the waves are said to be in phase(i.e. they have a phase difference of zero.)

If a crest and a trough are together, the waves are completely out of phase(i.e. they have a phase difference of π radians.

Suppose the wave moves from left to right and the particles at the origin O vibrate with simple harmonic motion.

The vibrations of the particle at P a distance x from the origin will be out of phase with vibration of the particle at O.

At a distance λ from O corresponds to a phase difference of 2π. Therefore the phase angle of Φ at P is $\Phi = \frac{2\pi x}{\lambda}$.

The displacement of any particle a distance x from O is given by $y = a\sin(\omega t - \Phi)$. Where Φ is the phase angle.

Hence $y = a\sin\left(\omega t - \frac{2\pi x}{\lambda}\right) = a\sin\left(2\pi ft - \frac{2\pi x}{\lambda}\right) = a\sin 2\pi\left(ft - \frac{x}{\lambda}\right)$

$y = a\sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$ which is the general equation for a progressive wave.

The quantity $k = \frac{2\pi}{\lambda}$ is called the wave vector or wave number.

Hence $y = a\sin(\omega t - kx)$

The negative sign in the equation indicates that vibrations at a point like P, to the right of O will lag behind those at O, for a wave traveling from left to right.

A wave traveling from the right to the left arrives at O before O. hence the vibrations at P would lead that at O. Hence $y = a\sin(\omega t + kx)$

Question

1. What is the phase difference between two waves of wavelength 12cm when one leads with other by (i) 6cm (ii) 9cm (iii) 3cm (iv) 12cm (iv) 36cm (v) 39cm

In all above use $\Phi = \frac{2\pi x}{\lambda}$

2. The displacement of a particle on a progressive wave is $y = 2\sin 2\pi(0.25x - 100t)$

Where x and y are in cm and t is in seconds. Calculate (i) wavelength ($\lambda = 0.04$m)

(ii) velocity of propagation of wave. (v = $4ms^{-1}$)

Compare with the general equation of a progressive wave

3. The displacement of a given wave traveling in the X – direction at the time t is

$y = a\sin 2\pi\left(\frac{t}{10} - \frac{x}{2}\right)$m. Find (i) velocity of the wave

(ii) period of the wave.

4. Find the speed of a compression wave in an iron rod of density $7.7\text{x}10^3 kgm^{-3}$ and whose young's modulus $2\text{x}10^{11}$Pa. (Use $v = \left(\frac{Y}{\rho}\right)^{\frac{1}{2}}$) (v = $5096.5ms^{-1}$)

5. A certain string has linear mass density of $0.25kgm^{-1}$ and is stretched with tension of 25N. One end is given a sinusoidal motion with frequency 5Hz and amplitude 0.01m. At the time t = 0, the end has zero displacement and is moving in the positive y – direction.

a) Find the wave speed, angular frequency, period, wavelength and wave number.

b) Write a wave function describing the wave.

c) Find the position of the point at x = 0.5m at the time t = 0.1s.

Transmission of energy by a wave.

In all progressive waves, energy propagates through the medium in the direction in which the wave travels. Each particle of the medium has energy of vibration and passes energy onto the next particle. In simple harmonic motion, where there is no damping, the energy of vibrating particle changes form kinetic energy to potential energy and back, with the total energy, E, remaining constant.

E = maximum kinetic energy

$E = \frac{1}{2}mv_{\max}^2$, but $v_{\max} = \omega A$

Where A is the amplitude of vibration

Hence $E = \frac{1}{2}mv_{\max}^2 = \frac{1}{2}m\omega^2 A^2$

Also $\omega = 2\pi f$, where *f* is frequency of vibration.

$$E = \frac{1}{2}m \times \left(4\pi^2 f^2\right) \times A^2 = 2m\pi^2 f^2 A^2$$

As a wave passes through a medium, the energy per unit volume of the medium is the energy per particle times the number of particles n per unit volume.

There fore the energy per unit volume $= \frac{2m\pi^2 f^2 A^2}{Volume} = 2\rho\pi^2 f^2 A^2$

Where $\rho = \frac{m}{Volume}$ = density of the wave particles.

Intensity (I) of the wave is the energy transfer per unit time per unit area perpendicular to the direction of propagation of the waves.

$$I = \frac{E}{area \times time} = \frac{2m\pi^2 f^2 A^2}{area\ (a) \times time\ (t)}$$

But $m = \rho \times volume = \rho \times al$

Where *a* is cross section area, *l* is length.

$$I = \frac{2\rho al\pi^2 f^2 A^2}{a \times t}$$

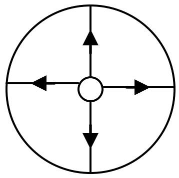
But $\frac{l}{t} = v$

Hence $I = 2\rho v\pi^2 f^2 A^2$

Example

The speed of sound in air is 330ms^{-1}. A source of sound of frequency 300Hz radiates energy in all directions at a rate of 10W. Find (i) the intensity of sound at a distance of 20m from the source.

(ii) the amplitude of sound wave at this distance.
(density of air at s.t.p = 1.29kgm^{-2})



$$I = \frac{power}{area}$$

But area $= 4\pi r^2 = 4\pi \times 20^2 = 1600\pi$

$$I = \frac{10}{1600\pi} = \frac{1}{160\pi} Wm^{-2}$$

(ii) Intensity $I = 2\rho v \pi^2 f^2 A^2$

Hence

$$A^2 = \frac{I}{2\rho v \pi^2 f^2} = \frac{1}{160\pi} \div 2\rho v \pi^2 f^2 = \frac{1}{160\pi^3 \times \left(2 \times 1.29 \times 330 \times 300^2\right)} = 1.621 \times 10^{-6} m^2$$

A = …………m

Principle of Superposition of waves

The resultant displacement at any point is the sum of the separate displacement s due to the two waves.

Let y_1 and y_2 represent the displacement of the individual waves in the planes.

When a crest falls on a crest or a trough falls on a trough, the resultant amplitude is double the amplitude of one wave.

When a crest combines with a trough, the resultant amplitude is zero.

Stationary waves

Stationary waves are due to superposition of two progressive waves having the same speed and frequency and nearly equal amplitude but traveling in opposite directions.

Consider two waves where y_1 and y_2, where $y_1 = a\sin(\omega t - kx)$. Hence $y_2 = a\sin(\omega t + kx)$

The resultant displacement $y = y_1 + y_2$

$$y = a\sin(\omega t - kx) + a\sin(\omega t + kx) = 2a\sin\omega t \times \cos kx = (2a\cos kx)\sin\omega t$$

$$y = (2a\cos kx)\sin\omega t = A\sin\omega t$$

Where $A = (2a\cos kx)$

The amplitude A is maximum and is equal to 2a at x = 0, x = λ/2, x = λ and so on. These points are thus antinodes and hence separation between consecutive antinodes is λ/2.

The displacement is zero when x = λ/4, x = 3λ/4, x = 5λ/4. and so on. These points are called nodes and hence they are midway between consecutive antinodes.

Note: Within a stationary wave, there is no flow of energy through a medium.

There is energy of motion between each vibrating segment but this energy is not transferred across node and is stationary.

Difference between progressive and stationary waves

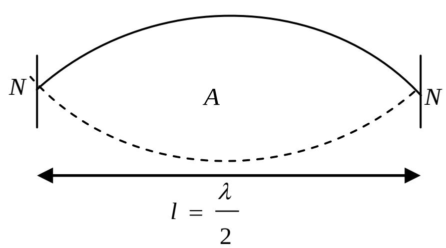
Stationary wave	***progressive***
All particles between successive nodes, have their vibration are in phase.	The phase of vibration of points near each other are all different
All points along the wave vibrate with the same amplitude.	Each points along the wave has a different amplitude i.e. Amplitude = 2acosθ

Stationary wave in strings

Modes of vibration

The ends of a stretched string are fixed and there fore the ends of the string must Be the displacement nodes .if the string is displaced in the middle, a stationary wave is formed

First Harmonic (fundamental)



The wave formed in this case is the simplest form of vibration and is called the fundamental .

The frequency at which it vibrates is called the fundamental frequency.

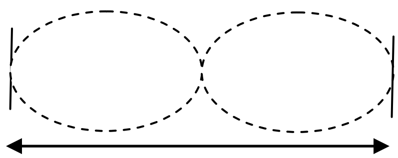
If f is the frequency (Fundamental frequency). Then $f_1 = \left(\frac{v}{\lambda}\right)$

But $\lambda = 2l$

$f_1 = \left(\frac{v}{2l}\right)$ where v is the speed of the wave.

Second Harmonic (first Overtone).

When the wave is plucked quarter way from one end, the wave formed is shown below;

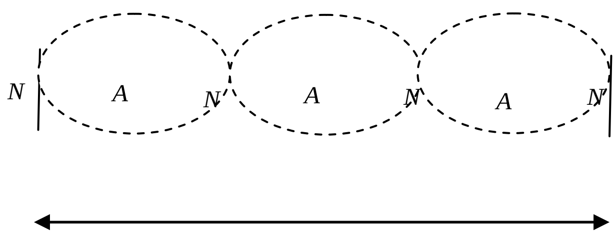


$l = \lambda$

If $\boldsymbol{f_2}$ is the frequency of the wave;

$$f_2 = \frac{v}{\lambda_2} = \frac{v}{l} = \frac{2}{2} \times \frac{v}{l} = 2 \times \left(\frac{v}{2l}\right) = 2f_1$$

Third harmonic (2^nd^ overstone)



$l = 3 \times \frac{\lambda}{2}$

$$f_3 = \frac{v}{\lambda_3} = \frac{v}{\left(\frac{2}{3}l\right)} = 3 \times \frac{v}{2l} = 3f_1$$

Frequencies which are higher than fundamental frequency are called overtones.

Waves formed by a stretched string are of the frequencies $f_{1,}\ 2f_{1,}\ 3f_1, 4f_1$ ..

If $\frac{\lambda_n}{2} = \frac{l}{n} = \lambda n = \frac{2l}{n}$,

The frequency fn of the n^th^ harmonic is given by :-

$f_n = \frac{v}{\lambda n} = \frac{v}{\left(\frac{2l}{n}\right)} = \frac{nl}{2l}$.......... ....$(i)$ but when $n = 1$, $f_1 = \frac{v}{2l}$ hence

$$f_n = nf_1$$

The frequency of the various overtones is whole number multiples of the fundamental frequency.

Note: An overtone of any frequency above the fundamental frequency.

A harmonic is an integral value of the fundamental frequency.

Recall, the speed of the vibrating string is $v = \sqrt{\frac{T}{\mu}}$

Where ***T*** is the tension in string
μ is the mass per unit length.

From eqn.(i) $f_n = \frac{n}{2l}\sqrt{\frac{T}{\mu}}$, where n = 1, 2, 3, 4, ……….

Examples

1.A wire of length 400mm and mass 1.2gm is under a tension of 120N.

(i) what is the fundamental frequency of vibration.

(ii) The frequency of the 3rd harmonic

(i) $f_1 = \frac{1}{2l}\sqrt{\frac{T}{\mu}} = \frac{1}{\left(2 \times 400 \times 10^{-3}\right)}\sqrt{\frac{120}{\left(\frac{1.2 \times 10^{-3}}{400 \times 10^{-3}}\right)}} = 250\ Hz$

(ii) $f_3 = 3f_1 = 3 \times 250 = 750\ Hz$

2. The mass of the vibrating length of sonometer wire is 1.20gm and it is found that a note of frequency 512 Hz is produced when wire is sounding its second overtone .If the tension of the wire is 100N, calculate the vibrating length of wire.

$f_3 = 3f_1$

512 = $3f_1 = \frac{3}{2l}\sqrt{\frac{T}{\mu}} = \frac{3}{2l}\sqrt{\frac{100}{\frac{1.2 \times 10^{-3}}{l}}}$

l = 0.715m

Question

A plane string 1.5m long is made of steel of density $7.7 \times 10^3 kgm^{-3}$ and Young's modulus $2 \times 10^{11} NM^{-2}$. It is maintained at a tension which produces an elastic strain of 1% in the string. What is the fundamental frequency of the transverse vibration of the string

Experiment verification $f_1 = \frac{1}{2l} \times \sqrt{\frac{T}{\mu}}$

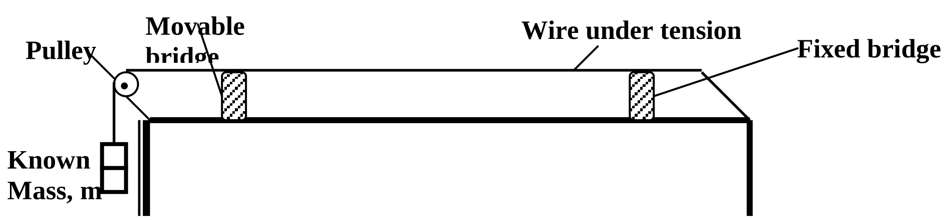
These frequency $f_1$of the fundamental mode of vibration is given by $\frac{1}{2l} \times \sqrt{\frac{T}{\mu}}$**,** it follows that

(i) $f_1 \propto \frac{1}{l}$ (T and μ constant)

(ii) $f_1 \propto \sqrt{T}$ (*l* and μ constant)

(iii) $f_1 = \sqrt{\frac{1}{\mu}}$ (T and *l* constant)

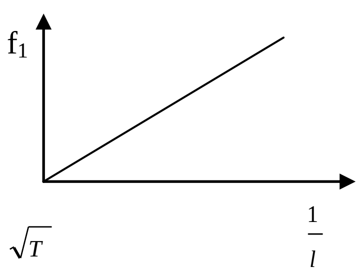
These relationship are sometimes referred to as the laws of vibration of a stretched string. They may be verified experimentally by using a son meter describe below:



(i) $f_1 \propto \frac{1}{l}$

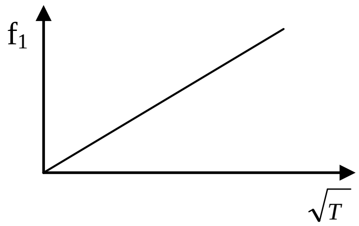
Having select suitable values of T and μ, the movable bridge is adjusted that the vibration length *l* of the wire produced the same note as the tuning fork of known frequency .The procedure is repeated using tuning fork of other known frequency without altering T and μ .

A graph of f_1 against $\frac{1}{l}$ is plotted and is linear passing through the origin.



(ii) $f_1 \propto \sqrt{T}$

With *l* kept constant at the same suitable value, the mass, m and there fore tension T is constant so that when the wire is plucked, it produces the same note as a tuning fork of known frequency, f_1. The procedure is repeated using tuning forks of other known frequencies without altering *l* and μ. A graph of f_1 against $\sqrt{T}$ is plotted and is linear, passing through the origin.

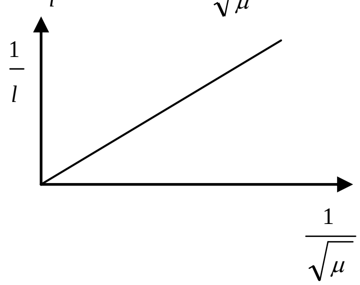


Hence $f_1 \propto \sqrt{T}$

(iii) $f_1 \propto \frac{1}{\sqrt{\mu}}$

The mass per unit length μ is determined by weighing. The length *l* of the wire is then adjusted so that the wire is plugged, it produces the same note as one of the turning forks. The procedure is repeated with wires of different masses per unit length, each wire must be under the same tension as the first wire and in each case the length is adjusted until the wire vibration of the same frequency as the tuning fork that was used of the first wire .

A graph of $\frac{1}{l}$ against $\frac{1}{\sqrt{\mu}}$ is plotted and is linear through the origin.



The graph shows that $\frac{1}{l} \propto \frac{1}{\sqrt{\mu}}$ and since $f_1 \propto \frac{1}{l}$ hence $f_1 \propto \frac{1}{\sqrt{\mu}}$.

Longitudinal stationary wave in pipes

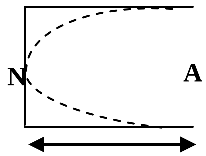
Closed pipes.

This consist essentially of a metal pipes closed at one end and the other.

open boundary condition.

At the closed end, there is a displaced node at the open end here is displaced antinodes The allowed oscillation nodes or standing wave patter are:-

(i) ***Fundamental note***



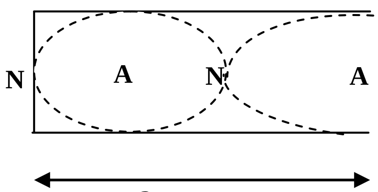
$$l = \frac{\lambda_1}{4}$$

Fundamental frequency, $f_1 = \frac{v}{\lambda_1} = \frac{v}{4l}$ ……………….(i)

Fundamental lowest frequency (f_1)

It is obtained when the simplest stationary wave form is obtained

(ii) ***First overtone*** (3rd harmonic)

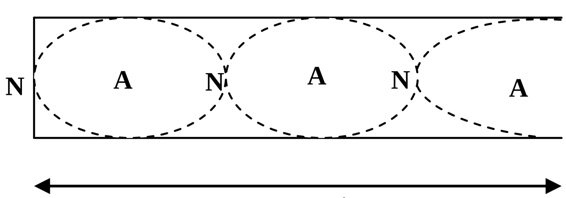


$$l = \frac{3}{4}\lambda_3$$

Frequency of first overtone f_3 is given by

$$f_3 = \frac{v}{\lambda_3} = \frac{v}{\left(\frac{4l}{3}\right)} = \frac{3v}{4l} = 3 \times \left(\frac{v}{4l}\right) = 3f_1$$

(iii) ***Second overtone*** (5th harmonic)



$$l = \frac{5\lambda_5}{4}$$

$$f_5 = \frac{v}{\lambda_5} = \frac{v}{\left(\frac{4l}{5}\right)} = \frac{5v}{4l} = 5f_1$$

The frequencies obtained with a closed pipe are f_1,$3f_1$, ,$5 f_1$, $7f_1$ $9f_1$,etc i.e. only odd harmonics' are obtainable .

In general ; $f_n = \frac{nv}{4l} = nf_1$ **,** n = 1, 3, 5, 7………

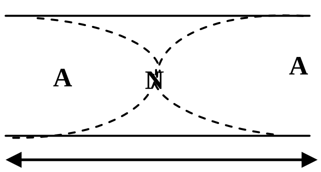
Open pipes

Pipes which are open at both ends.

Boundary conditions

Antinodes are at both ends

(i) ***fundamental mode .***

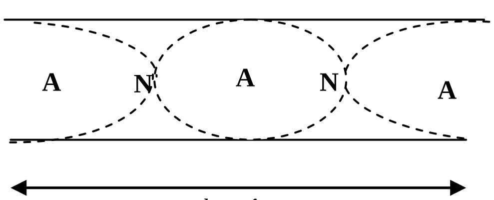


$$l = \frac{\lambda_1}{2}$$

Fundamental frequency

$$f_1 = \frac{v}{\lambda_1} = \frac{v}{2l}$$

(ii***) first overtone (second harmonic)***



$$l = \lambda_2$$

$$f_2 = \frac{v}{\lambda_2} = \frac{v}{l} = \frac{2}{2} \times \frac{v}{l} = 2 \times \left(\frac{v}{2l}\right)$$

$$f_2 = 2f_1$$

Thus frequencies for notes produced by open pipes are $f_1, 2f_1, 3f_1, 4f_1$……….. ..

In general $fn = \frac{nv}{2l} = nf_1 \qquad n = 1,2,3,4........$

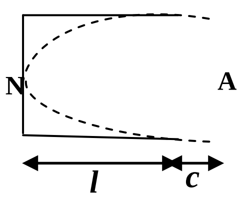
So an open pipe can produce both odd and even harmonic

End correction

The at the open end of the pipe is free to move and hence the vibration at this end of the sounding pipe extend a little into the air outside .

Antinodes of the stationary were due to any note is in practice a distance ***c*** from the open end. The distance ***c*** is known as the end correction.

For the closed pipe;-



$$l + c = \frac{\lambda_1}{4}$$

$$f_1 = \frac{v}{\lambda_1} = \frac{v}{4(l + c)}.$$

For open pipe ;-

Fundamental mode,

$$l + 2c = \frac{\lambda_1}{2}$$

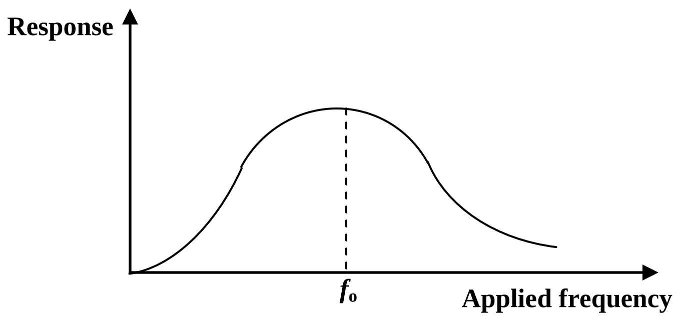
Fundamental frequency, $f_1 = \frac{v}{\lambda_1} = \frac{v}{2(1 + 2c)}$

Resonance in pipes.

Any force oscillating system (air column, mechanical system ,diving board)gives a maximum response when the diving frequency ***f***, is equal to the natural frequency $\boldsymbol{f_o}$ of the forced system

The system is said resonate when this happens.

Sketch of response against frequency .

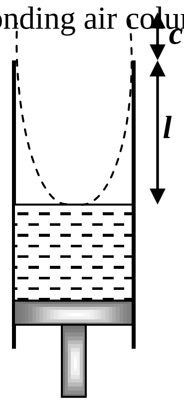


Resonance occurs when a particular body or system is set into oscillation at its own natural frequency as a result impulses received from other systems vibrating with the same frequency .

If the prongs of a tuning fork are held over the top of the pipe , air inside is set in vibration by the periodic force extended on it by the prongs .The vibration are feeble as they are forced vibrations and the intensity of the sound is correspondingly small. But when a tuning fork of the same frequency as the fundamental frequency of the pipe is held over it , the air inside is set into resonance by periodic force and the amplitude of vibration is loud . A loud note of the same frequency as the note is heard coming from the pipe . In general, for a tube of varing length , resonance is obtained for some lengths where a stationary wave is set up with an antinode at one end and node at the closed end .

Measurement of velocity of sound using a resonance tube.

If a sounding tuning fork is held over the open end of a tube T filled with water, resonance is obtained at the same positions as the water level is lowered . The first two resonance length are obtained. The corresponding air column lengths l_1 and l_2 are measured.



At first resonance $l = l_1$. Hence $\frac{\lambda}{4} = l_1 + c$ ……………….(i)

At second resonance $l = l_2$. Hence $\frac{3\lambda}{4} = l_2 + c$ ……………..(ii)

From equation (i) and (ii) $\lambda = 2(l_2 - l_1)$
From $v = f\lambda$
Hence $v = 2 \times f(l_2 - l_1)$

Examples

1 A progressive and stationary wave each have a frequency of 240Hz and a speed of $80ms^{-1}$

(i) calculate the phase difference between two vibrating points in the progrese wore when they are 6 cm apart.

$f = 240\ Hz$, $v = 80\ ms^{-1}$

$\lambda = \frac{v}{f} = 80/240$

Phase difference $\sigma = \frac{2\pi\chi}{\lambda} = \frac{2\pi \times 6 \times 10^{-2} \times 240}{80} = 0.36\pi$ rad.

(ii) distance between nodes in the stationary wave .

$\lambda = \frac{v}{f} = 80/240 = \frac{1}{3}m$

Hence distance between nodes = $\frac{\lambda}{2} = \frac{1}{3} \times \frac{1}{2} = \frac{1}{6}m$

1. A plane progressive wave is given by $y = a\sin\left(100\pi t - \frac{10}{9}\pi x\right)$

(i) Write the equation of a progressive wave which would give rise to the stationary wave if superimposed on the above .

$y = a\sin\left(100\pi t + \frac{10}{9}\pi x\right)$

(ii) Find the equation of the stationary and hence determine the amplitude of vibration

$y_1 = a\sin\left(100\pi t - \frac{10}{9}\pi x\right)$

$$y_2 = a\sin\left(100\pi t + \frac{10}{9}\pi x\right)$$

Using the principle of superposition, resultant displacement $y = y_1 + y_2$

Hence $y = a\sin\left(100\pi t - \frac{10}{9}\pi x\right) + a\sin\left(100\pi t + \frac{10}{9}\pi x\right)$

$$y = 2a\sin(100\pi tx) \times \cos\left(\frac{10}{9}\pi x\right)$$

Hence Amplitude = $2a\cos\left(\frac{10}{9}\pi x\right)$

(i) Determine the velocity and frequency of the stationary wave.

$f = \frac{I}{T}$ where $T = \frac{2\pi}{\omega}$

f = $\frac{\omega}{2\pi} = \frac{100\pi}{2\pi} = 50\ Hz$

velocity of the wave

$$k = \frac{10}{9}\pi = \frac{2\pi}{\lambda}$$

$$\lambda = 1.8mm = 1.8 \times 10^{-3}\ m$$

$$v = f\lambda = 50 \times 1.8 \times 10^{-3} = 0.09\ ms^{-1}$$

4 A glass tube open at the top is held vertically and filled with water. A tuning fork vibrating at 264 Hz is held above the table and water is allowed to flow out slowly .The first resonance occurs when the water level is 31.5cm from the top while the 2nd resonance occurs when the water level is 96.3cm from the top

Find;-

(i) Speed of sound in the air column.

At first resonance, $\frac{\lambda}{4} = l_1 + c$

$$\frac{\lambda}{4} = 0.315 + c.......... ....(i)$$

At second resonance, $\frac{3\lambda}{4} = 0.963 + c.........(ii)$

Equation (ii) – (i) you get $\frac{\lambda}{2} = 0.963 - 0.315$

$\lambda = 2 \times (0.963 - 0.315) = 1.296\ m$

$v = f\lambda = 264 \times 1.296 = 342.144\ ms^{-1}$

(ii) End correction.

$\frac{\lambda}{4} = 0.315 + c$

$c = \frac{\lambda}{4} - 0.315 = \frac{1.296}{4} - 0.315 = 0.324 - 0.315 = 0.009\ m$

Exercise

1. A stretched wire of length 0.75m, radius 1.36mm and 1380kgm^{-3} is dumped to both sides and is plucked in the middle .The fundamental note is produced by the wire has the same frequency as the first overtone in the pipe of length 0.15m closed at one end .

(i) Sketch the standing wave pattern in the wire.

Calculate the tension in the wire (V of sound in air =330MJ) $\left[v = \sqrt{\frac{I}{\mu}} \right]$

2. A wave of amplitude 0.2m, wave length 2m and frequency 50Hz , propagating in the X-direction . If the initial displacement is 0 at pt x = 0. Write the expression of the displacement of the wave at any time.

(1) Find the speed of the wave.

(b) 2 waves of frequency 256Hz respectively travel with a speed of 340m +6.4a medium. Find the phase difference of an amplitude point 2m from where they are initially in phase.

(ii) Design an experiment to demonstrate that a metal wire under tension can vibrate with more than one frequency

3. Give the factors that affect the frequency of the transverse wave traveling along a and how the frequency varies with each factor.

(b)A string of strength 31.6cm of fixed at both ends so that it is taut. The lowest frequency of the transverse were it can produce is 880Hz .Calculate the speed of wave.

(c) A long glass tube is filled with water. A tuning fork is held at the mouth of the tube and the tube is gradually emptied. Explain what happens.

4. A small speaker emitting a note of 250Hz is placed over the open upper end of a vertical tube when it is full of water. When the water is gradually run out of the tube, the air when it is 0.98m below the top column resonates, initially when the water surface is below 0.310 below the top. Find V and end correction.

5. State the conditions that lead to the establishment of the standing wave.

(ii) A uniform tube 50cm long stands vertically with its lower end dipping into the water. The tube resonates to a tuning fork of frequency 256Hz when its length above water is 12cm and again when it is 39.6cm. Eliminate the lowest freq. to which the tube resonates when it is open at both ends.

(iii) A wave of mass $1.0x\ 10^{-2}$kg and decimeter $6.0x10^{-4}$m is stretched between rigid supports 1.0m apart.
The tension in the string is 60N. Find the change in the freq. of the fundamental rock when the temperature of the wire is lowered by 100K given that the speed

$$v = \sqrt{\frac{t}{\mu}}$$

Young's modulus $2.0x10^{11}$pa.
Linear expansion $1.5x10^{-6}K^{-1.}$

Beats

When two notes with slightly different frequencies but equal amplitude are sounded together , they interfere with one anther , and the resultant effect in the sound where budnen increases and decreases periodically i.e maximum sound alternating with minimum sound is high . This phenomenon is known as *beats*.

The frequency of beats is the number of intense sounds heard per second. The variations in amplitude (and intensity)are called beats. The number of times the sound readies maximum intensity per second is called the *beat intensity.*

The production beats is a wave effect explained by the principle of super position. Beats are due to interference in time because the sources are not coherent (they are of different frequency), there is sometimes *reinforcement* at a given time and at other times *cancellation*(amplitude is zero), Suppose the beat period (i.e time between 2 successsive maximum) is T , and that one wave train of frequency f_1 , makes one cycle more than that of frequency f_2.

Number of cycles of frequency f_1, = f_1T

and number of cycles of frequency f_2 =f_2T

$$f_1\mathrm{T} - f_2\mathrm{T} = 1$$

$$f_1 - f_2 = \frac{1}{\mathrm{T}}$$

But $\frac{1}{\mathrm{T}} = f$ (the beat frequency).

Hence $\left|f_1 - f_2\right| = f$

Uses of beats.

1. Beats are used to tune an instrument to a given note. As the instrument note approaches a given note, beats are heard. The instrument may be regarded as tuned when beats occur at a very slow rate.

2. To measure frequency f_1 of a given note. A note of known frequency f_2 is used to provide beats with the unknown note and the frequency f is obtained by counting the number beats made in a given time. Hence $\left|f_1 - f_2\right| = f$. To decide which value of f_1 is correct, the end of the unknown tuning fork prong is loaded with a small piece of plasticine which diminishes the frequency a little and the two notes are sounded together. If the beat frequency increases, then f_2 is greater than f_1. If it decreases, then $f_1 \rangle f_2$.

Examples

1. A tuning fork of unknown frequency and a standard fork of 440Hz are sounded simultaneously and beats of frequency 4Hz are heard. What deduction can you make regarding the frequency of the unknown fork?

$\left|f_1 - f_2\right| = f$

Hence $f_1 = f + f_2 = 440 + 4 = 444 \ Hz$ or $f_1 = f_2 - f = 440 - 4 = 436 \ Hz$

(ii) A small piece of wax is attached to the prongs of the unknown fork and in between forks are sounded again. It is found that the beat frequency is now 3Hz. What deduction can you make, explain your reason.

Since the beat frequency decreases from 4Hz to 3Hz, this implies that $f_1 \rangle f_2$

Hence $f_1 = 444 \ Hz$

2. Beats are produced by a plucked stretched wire and a resonance tube closed at one end each sounding at its fundamental note. The air column has a length of 0.168m the end correction being 0.012m. The wire has a vibrating length of 0.27m and is under tension of 100N. The mass of the position of the wire is 4x10^{-4}kg.

(i) Calculate the frequency of the beats heard. If velocity of sound in air column is 350ms^{-1}.

For the closed tube, $\frac{\lambda}{4} = l_1 + c$

$$\lambda = 4 \times (l_1 + c) = 4 \times (0.168 + 0.012) = 0.72 \ m$$

$$f_1 = \frac{v}{\lambda} = \frac{350}{0.72} = 486.1 Hz$$

For the string, $f_2 = \frac{1}{2l} \times \sqrt{\frac{T}{\mu}} = \frac{1}{2 \times 0.27} \times \sqrt{\frac{100}{\left(\frac{4 \times 10^{-2}}{0.27}\right)}} = 481.1 Hz$

Beat frequency, $f = f_1 - f_2 = 486.1 - 481.1 = 5 Hz$

(ii) Calculate the change in tension of the wire that would make the frequencies of the two notes the same.

Using f = 486.1Hz

$$f = \frac{1}{2l} \times \sqrt{\frac{T}{\mu}} = 486.1 Hz$$

$$\frac{1}{2 \times 0.25} \times \sqrt{\frac{T}{\left(\frac{4 \times 10^{-4}}{0.27}\right)}} = 486.1$$

Hence T = 102N

Hence change in tension = 102 – 100 = 2N.

Questions

1. Two tuning forks X and Y are sounded together to produce beats of frequency 8Hz. Fork X has a known frequency of 512Hz. When Y is loaded with a small plasticize beats at frequency of 2Hz are heard when the two tuning forks are sounded together.

Calculate the frequency of Y when unloaded. (520Hz)

2. The wire of a sonometer of mass per unit length $10^{-3}kgm^{-1}$ is stretched on the two bridges by a load of 40N. When the wire is struck at the center point so that it executes its fundamental vibration, and at the same time a tuning fork of 264Hz is sounded and beats are heard and found to have a frequency of 3Hz. If the load is slightly increased, the beat frequency is lowered. Calculate the separation of the standing wave.($l = 0.375m$)

Doppler's effect.

The pitch of the note from the siren (whistle) of a first traveling ambulance or train appears to a stationary observer to drop suddenly as 16 panes. This apparent change in the frequency of a wave motion when there is relative motion between the source and observer is called the *Doppler Effect.* It occurs with electromagnetic waves and sound waves.

Doppler Effect in sound waves.

The following symbols will be used i.e,

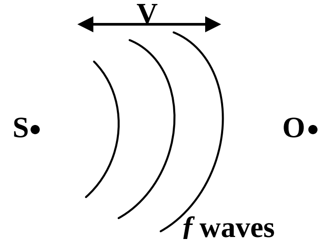
V = speed of sound in air

u_s = speed of source, S

u_o = speed of observer, O

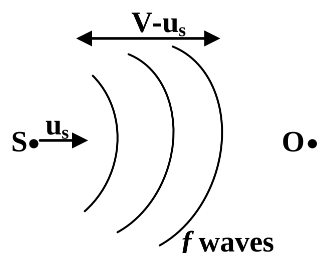
f = frequency of sound waves.

(a)Stationary source and observer.



f waves emitted per second occupy a distance, V and hence the wave length $\lambda = \frac{V}{f}$

b) A stationary observer and a source moving towards the observer.



f waves are compressed into a smaller distance $V - u_s$ per second

The apparent wave length $\lambda' = \frac{V - u_s}{f}$

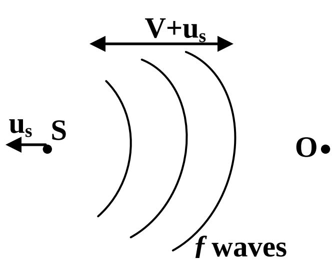
Apparent frequency, f_1 = <u>velocity of waves relative to O.</u>

Apparent wave length, λ'.

Therefore, $f_1 = \frac{V}{\left(\frac{V - u_s}{f}\right)} = \frac{Vf}{V - u_s}$, hence $f_1 \rangle f$.

If the source is moving with uniform speed, the pitch of the note is constant but *higher* than the true pitch.

c) Source moving away from a stationary observer



Apparent wave length, $\lambda_2 = \dfrac{V + u_s}{f}$

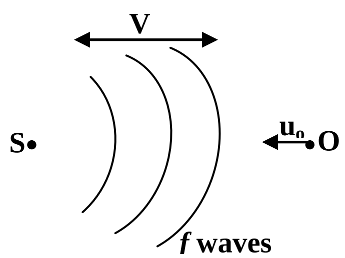
The apparent frequency, $f_2 = \dfrac{V}{\lambda_2} = \dfrac{V}{\left(\dfrac{V + u_s}{f}\right)} = \dfrac{Vf}{V + u_s}$.

$f_2 < f$ (i.e. observer hears a note of lower pitch than the true pitch).

Apparent change in frequency heard by the observer as the source passes = f_1 - f_2

$$= \frac{Vf}{V - u_s} - \frac{Vf}{V + u_s} = \frac{2Vu_s f}{V^2 - u_s^2}$$

d) Observer moving to a stationary source.

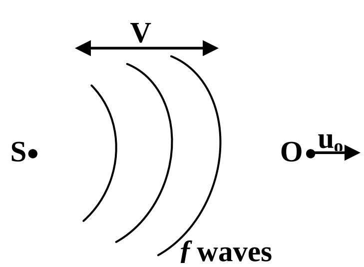


f waves occupy a distance V, since the source is stationary. The velocity of the wave relative to O is V + u_o. Apparent frequency of the wave f_1 = V + u_o but $\lambda = \dfrac{V}{f}$

$$f_1 = \frac{V + u_o}{\left(\dfrac{V}{f}\right)} = \frac{(V + u_o)f}{V}.$$

$f_1 < f$. (Implying that pitch is higher than the true pitch).

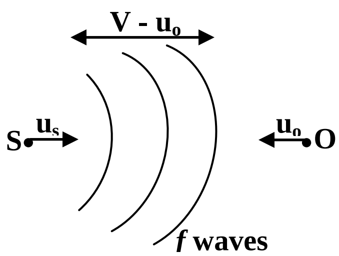
e) Observer moving away from a stationary source.



Apparent frequency, $f_1 = \dfrac{V - u_o}{\left(\dfrac{V}{f}\right)} = \dfrac{(V - u_o)f}{V}$.

$$f_2 < f$$

f) Source and Observer moving towards each other.



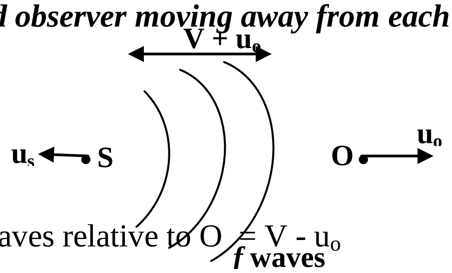
f waves occupy a distance V-u_s. The wave length of the waves reaching O is $\lambda' = \dfrac{V - u_s}{f}$.

The velocity of the relative to O is V + u_o

Apparent frequency, $f_1 = \dfrac{V + u_o}{\lambda'} = \dfrac{V + u_o}{\dfrac{V - u_s}{f}}$

$$f_1 = \frac{f(V + u_o)}{V - u_s}$$

g) Source and observer moving away from each other.



Velocity of waves relative to O = V - u_o

Hence apparent frequency $f_2 = \dfrac{V - u_o}{\lambda'} = \dfrac{(V - u_o)f}{V + u_s}$

In general, apparent frequency f_1 is given by

$$f_1 = \frac{V - u_o}{\lambda'} = \frac{(V \pm u_o)f}{V \mp u_s}$$

Upper sign applies to approach.

Lower sign applies to moving away from each other.

Application of Doppler Effect in Light.

a*) Measurement of the speed of a star.*

Consider a star moving with a velocity u_s away from the earth and which emits light of wave length λ . Let f be the frequency of the light. Let c be the velocity of light in vacuum. Owing to the motion of the star, the f waves emitted per second by the source occupy a distance equal to $c + u_s$

The apparent wave length λ', to an observer on earth in line with the star's motion is

$$\lambda' = \frac{c + u_s}{f} = \frac{c + u_s}{\frac{c}{\lambda}} = (1 + \frac{u_s}{c})\lambda$$

The shift in wave length is $\Delta\lambda = \lambda' - \lambda = \frac{u_s \lambda}{c}$

Thus $\lambda' > \lambda$ is when star is moving away from the earth, i.e there is a shift towards the red end of the visible spectrum (RED SHIFT).

Suppose the star is moving towards the earth with velocity, u_s the f waves occupy a distance $(c - u_s)$

The apparent wave length λ', to an observer on earth in line with the star's motion is

$$\lambda' = \frac{c - u_s}{f} = \frac{c - u_s}{c/\lambda} = \left(1 - \frac{u_s}{c}\right)\lambda$$

$$\Delta\lambda = \lambda - \lambda' = \frac{u_s \lambda}{c}$$

There is a shift towards the blue end of the visible spectrum (BLUE SHIFT).

The Doppler shift can be used to measure the speed of a star. A photograph of the star is taken. The spectral lines ((λ') are compared with the corresponding lines (λ) obtained by photographing in the laboratory, an arc or spark spectrum of an element known to be present in the star. If λ' is displaced towards the red end, the star is receding from the earth; if it is displaced towards the blue end, the star is approaching the earth.

The speed of the star is $u_s = \left|\frac{\lambda^1 - \lambda}{\lambda}\right| c.$

b) Measurement of plasma temperatures

At very high temperatures (~10^6K) molecules of the glowing gas are moving away and towards the observer with high speeds.

Owing to the Doppler Effect, the wave length λ of a particular spectral line is broadened. One edge of the line corresponds to an apparently decreased wave length λ_1 due to molecules moving towards the observer, and the other edge to an apparently increased wave length λ_2 due to molecules moving directly away from the observer. The line is broadened by an amount $\lambda_2 - \lambda_1 = \frac{2u_s}{c}\lambda$, Where u_s is the mean speed of the molecules.

The broadening $\Delta\lambda$ can be measured by a diffraction grating. Knowing λ and c, u_s can be calculated. The mean square speed of the molecules of the gas is $= \left(\frac{3RT}{M}\right)^{1/2}$ where T is the absolute temperature, R is the molar gas constant and M is the molar mass.

The temperature T can be estimated by equating u_s^2 to $\frac{3RT}{M}$ to yield $T = \frac{Mu_s^2}{3R}$.

Examples

1. A stationary observer notices the pitch of a police car changing the ratio of 4:3 when passing him. If the speed of sound is $350ms^{-1}$, calculate the speed of the car.

$$f_1 = \frac{Vf}{V - u_s}, \quad f_2 = \frac{Vf}{V + u_s}$$

$$\frac{f_1}{f_2} = \frac{4}{3}$$

$$3f_1 = 4f_2$$

$$3 \times \frac{350f}{350 - u_s} = 4 \times \frac{350f}{350 + u_s}$$

$$u_s = 50\ ms^{-1}$$

2. An observer moving between two identical sources of sound along the straight line joining them hears beats at the rate of $4s^{-1}$. At what velocity is he moving if the frequency of each source is 500Hz and the velocity of sound when he makes the observation is $340ms^{-1}$.

1st source moving towards observer $f_1 = \frac{(V + u_o)f}{V} = \frac{(340 + u_o) \times 500}{340}$

2nd source moving away from the observer $f_2 = \frac{(V - u_o)f}{V} = \frac{(340 - u_o) \times 500}{340}$

$f_1 - f_2 = 4$

$\frac{(340 + u_o) \times 500}{340} - \frac{(340 - u_o) \times 500}{340} = 4$

$u_o = 1.36\ ms^{-1}$

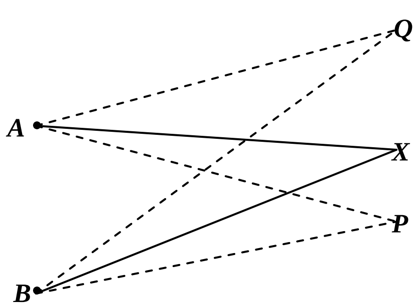
Questions

1. Two observers A and B are provided with sources of sound of frequency 500Hz. A remains stationary and B moves away from him at a velocity of $1.8ms^{-1}$. How many beats per second are observed by A and by B, the velocity of sound being $330ms^{-1}$? (2.73Hz)

2.

Interference of light waves.

Conditions for observable interference



Suppose two light sources A and B have exactly the same frequency and amplitude of vibration and their vibrations are always in phase with each other. Such sources are called coherent sources. *Coherent sources are those which emit light waves of the same wave length or frequency which are always in phase with each other or have a constant phase difference*.

Suppose X is equidistant from A and B, the vibration at X due to the two sources will always be in phase. The distance AX traveled by the wave originating at A is equal to the distance BX traveled by wave originating at B.

Assume that waves from A and B are moving at X have the same amplitude, then the amplitude of the resultant wave at X is twice that of either waves from A and B.

The light energy at X is proportional to the square of the amplitude of resultant wave and hence is four times that due to A or B alone. A bright beam of light is therefore obtained at X as A and B are coherent sources. It is due to constructive interference of light waves at A and B at X i.e a crest from A reaches at the same time as a crest from B.

Suppose Q is a point such that $BQ \rangle AQ$ by a whole number of wave lengths. The waves are moving at Q from A will be in phase as a wave moving at the same point from B, a bright band will be obtained at Q.

Therefore condition for constructive interference to occur at any point Y as such that the path difference BY –AY $= m\lambda$ where λ is light wavelength from sources A and B and m is an integer.

Consider a point P where distance from B is $1/2$ a wave length longer than its distance from A

i.e. AP –BP = $\frac{\lambda}{2}$ where λ is a wave length of light from A and B

Then the waves arriving at P from A will be out of phase with waves arriving at P from B. If the waves have equal amplitudes, we obtain the following:-

The resultant at P is zero as the displacements at any instant are equal and opposite of each other. No light is therefore seen at P. A dark band is obtained. Destructive interference is said to have occurred.

In general, if P is such that the path difference AP – BP $= (m + \frac{1}{2})\lambda$ where m is an integer, destructive interference is said to have occurred at P.

Optical path.

Suppose light travels a distance X in a medium of refractive index $\cap$. If λ is the wave length of light in the medium, the quantity $\sigma = \frac{2\pi x}{\lambda}$ is the phase difference due to the path.

If the speed of light in the median is V then the refractive index of the median $n = \frac{c}{\lambda}$ where

C = speed of light in vacuum. V = speed of light in medium.

But $c = \lambda_o f$, V =λf

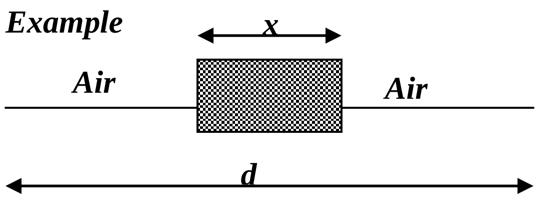
Where λo and λ are wave length in a vacuum and medium respectively.

$$n = \frac{\lambda_o f}{\lambda f} = \frac{\lambda_o}{\lambda}, \qquad \lambda = \frac{\lambda_o}{n}$$

$$\sigma = \frac{2\pi x}{\frac{\lambda_o}{n}} = \frac{2\pi n x}{\lambda_o}.$$

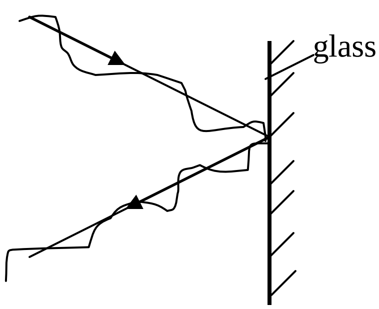
nx is called the optical path. It is the product of the refractive index and the length light covers in the medium.

Example



Optical path $= nx + 1 \times (d - x) = nx + (d - x)$

Phase difference on reflection

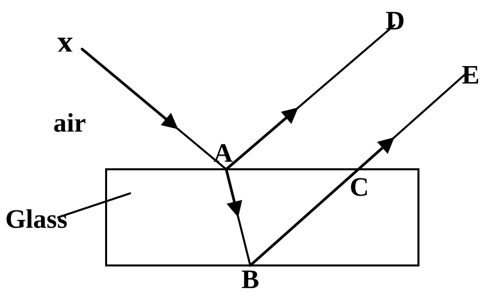


Light waves may undergo a phase change by reflection at some point in their path. If the waves are reflected at a denser medium e.g at the air –glass interface, a phase change of π radius compared to the incident waves occurs. This corresponds to a phase difference.

$$\sigma = \frac{2\pi\chi}{\lambda} \qquad \pi = \frac{2\pi\chi}{\lambda}$$

$$\chi = \lambda/2$$

Reflection occurs at the interface with a denser medium. No phase change in the less dense medium. Consider monochromatic light incident on a glass plate as shown.



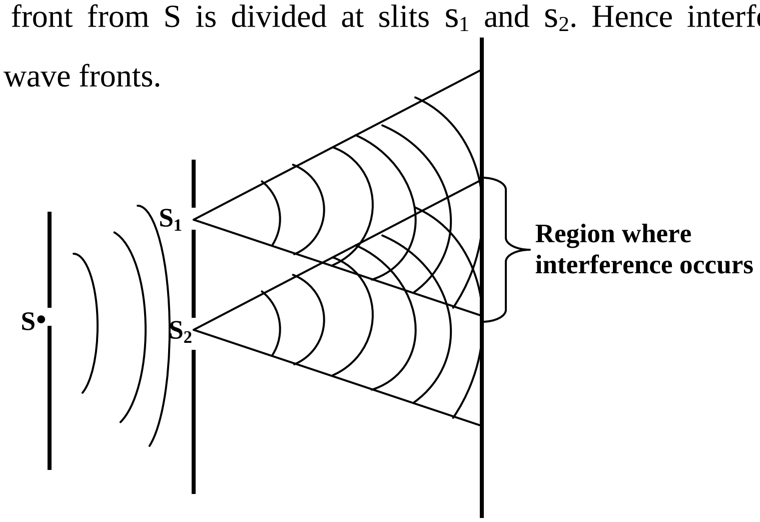
The optical path A to C is n (AB+BC).

There is no phase change at B since reflection occurs at an interface with a less dense medium but there is a phase change of π radius equivalent to a path of $\lambda/2$. When XA is reflected along AD. Therefore the optical path difference between light reflected at A and that reflected at B is n (AB+ BC) $= \frac{\lambda}{2}$.

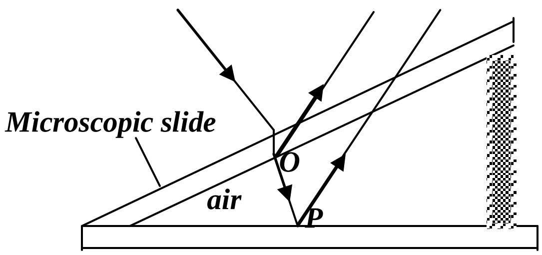
How to produce two coherent sources.

(i) *By division of wave front* e.g Young's double slit interference.

The wave front from S is divided at slits S_1 and S_2. Hence interference is occurring by division of wave fronts.

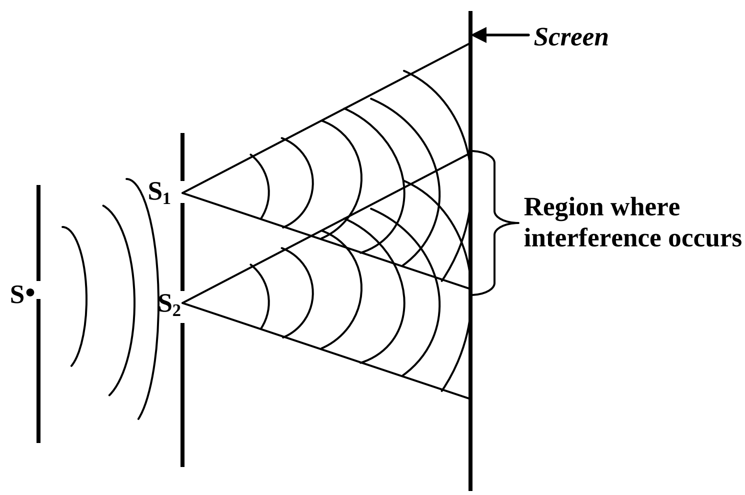


(ii) *Division of amplitude*. E.g air wedge.



Here some of the light falling on the wedge is reflected upwards from the bottom surface of the top side i.e at O and the rest which is transmitted through the air wedge is reflected upwards from the top surface of the bottom slide i.e at P so that the amplitude of the wave is divided into two parts.

Young's Double slit interference pattern.



Monochromatic light from a narrow vertical slit S falls on two other narrow slit S_1 and S_2. Which are very close together and parallel to S. S_1 and S_2 act as two coherent sources? Diffraction also takes place at S_1 and S_2. and interference occurs in the region where light from S_1 overlaps that from S_2. A series of alternate bright and bark equally spaced vertical bands (or fringes). Are observed on the screen.

Separation of fringe.

→ Qn. What would be the effect of replacing monochromatic light with white light in Young's double experiment?

Using white light, fewer fringes are seen and each colour produces its own set of friges which overlap. Only the central frige is white, its position being the only one where the path difference is zero for all colours. The first coloured fringe is bluish near to the central fringe and red at the far end.

The fringe spacing for red is greater than blue light. Therefore red light must have a greater wave length than blue light since $y \alpha \lambda$ (If a and are constant).